# On the Compton scattering vertex for massive scalar QED

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We investigate the Compton scattering vertex of charged scalars and photons in scalar quantum electrodynamics (SQED). We carry out its non perturbative construction consistent with Ward-Fradkin-Green-Takahashi identity (WFGTI) which relates 3-point vertices to the 4-point ones. There is an undetermined part which is transverse to one or both the external photons, and needs to be evaluated through perturbation theory. We present in detail how the transverse part at the 1-loop order can be evaluated for completely general kinematics of momenta involved in covariant gauges and dimensions. This involves the calculation of genuine 4-point functions with three massive propagators, the most non-trivial integrals reported in this paper. We also discuss possible applications of our results.

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#### I. INTRODUCTION

Evaluating Green functions for gauge theories in the non perturbative regime is as important as it is difficult. For example, understanding confinement and dynamical mass generation in quantum chromodynamics (QCD) requires knowledge of quark and gluon propagators in the infra red. These objects are tied to the 3- and 4-point vertices through Schwinger-Dyson equations (SDEs) as well as Slavnov-Taylor identities (STI), [1]. For example, unlike abelian gauge theories, 4-point ghost-ghost-quark-quark scattering kernel arises even at the level where these identities relate 2- and 3-point vertices in covariant gauges. 4-point quark-quark scattering kernel for off-shell external legs is another important quantity. Its infra red behaviour in the coordinate space would tell us how the strong potential rises for large distances between two static quarks. Study of the 4-gluon vertex can improve our insight of the running coupling in the infra red region, [2]. Understanding non perturbative behaviour of a 4-point function is a daunting task, so a detailed study of such a function in a relatively simpler scalar quantum electrodynamics (SQED) in terms of Ward-Fradkin-Green-Takahashi identity (WFGTI) [3] can provide an important first step towards much more involved similar functions in QCD.

We start by providing the most general basis in terms of which we can expand out the complete Compton scattering vertex. We use an appropriate set of basis vectors so that the components of the 4-point vertex longitudinal to the external photons can be readily identified. By invoking WFGTI, which relates the 4-point Compton scattering vertex to the 3-point scalar-photon vertex and the 2-point scalar propagator, we construct the longitudinal vertex to all orders in perturbation theory in terms of lower n-point functions. This WGTI-conserving construction will enable us to go to the next order of approximation in SDE studies. In principle, we shall now be able to truncate it at the level of the 4-point functions rather than the 3-point functions. Note that the WFGTI leaves the transverse part of the vertex undetermined which has to be evaluated using the brute force of perturbation theory.

Despite the fact that perturbative evaluation of n-point Green functions in gauge theories is receiving more attention, analytical results for arbitrary gauge and dimensions with completely off-shell external legs even for 1-loop 3-point vertices for SQED and QCD [4, 5, 6, 7] have been reported not so long ago. Results for spinor QED can be derived by appropriate replacement of color factors in the quark-gluon vertex. These perturbative results provide a natural guide to the possible non perturbative structures of the fermion-boson vertex, structures which are vital in the reliable truncations of Schwinger-Dyson equations at the level of the 3-point functions to study confinement, dynamical mass generation and hadron spectrum, see for example [8, 9, 10].

A natural next step in this direction is the evaluation of a 4-point function in arbitrary gauge and dimensions for off-shell external legs. To our knowledge, no such complete calculation exists for any gauge theory though a lot of work exists which deals with the calculation of box-diagrams in particular kinematical regimes related to the problem at hand. For example, a method to calculate massive Feynman integrals using Mellin-Barnes technique was developed in [11] for propagators and triangles with arbitrary powers of the denominators. The first on shell infra red divergent scalar box was calculated in [12] using a mass regulator for an internal photon; this result has extensively been employed in many electroweak and heavy quark calculations since then. Photon-photon scattering box diagrams were treated in [13] using different representations for 1-loop integrals. 2-, 3- and 4-point massive integrals for arbitrary momenta and dimensions were studied in [14] with explicit examples of the integrals relevant to Bhabha

scattering. The 4-point master integral in D dimensions using Mellin-Barnes technique was studied in [15]. General techniques for solving n-point scalar massive and massless 1-loop integrals were developed in [16, 17]. In [18], the box diagram in arbitrary dimension was calculated for on-shell external legs with internal propagators of varying masses in connection with the first order radiative corrections to Bhabha scattering in D dimensions. 1-loop amplitudes for 4-point functions with two external massive quarks and two external massless partons were studied in detail in [19]. In connection with  $2 \to 2$  QCD scattering amplitudes, scalar 1-loop integrals in different kinematical regimes have been evaluated in [20, 21, 22]. More recent work on the subject can be found in [23]. Our present article extends these earlier endeavours to all external legs being off-shell with completely different degrees of off-shellness. Moreover, instead of calculating one particular topology, we evaluate a complete process in a gauge theory (Compton scattering in SQED) in an arbitrary covariant gauge, calculating all the diagrams contributing to the process in general number of dimensions D.

SQED is a simple gauge theory as compared to spinor gauge theories (in the sense that there is no Dirac matrix structure), but the loop integrals have the same basic topologies as the ones that appear in spinor QED and QCD. This makes it an attractive theory. Recently, analytical result for the six-photon helicity amplitudes has been reported for SQED, [24]. Apart from the fact that we develop the machinery needed to obtain the off-shell 1-loop 4-point function, the study of processes such as Compton scattering

$$e^{-}(p) + \gamma(k) \rightarrow e^{-}(p') + \gamma(k') \tag{1}$$

has a value in its own right. For example, the electric and magnetic polarizabilities of a pion are related to the Compton scattering amplitude at threshold. Due to the fact that the internal structure of the pion contributes little to these polarizabilities, 1-loop results are important to calculate (see [25] and references therein) because these play a key role in the calibration of high energy colliders. Moreover, in terms of physical insight, the analysis of processes with virtual or real photons attached to an electron or a quark line help us probe the electron or nucleon substructure (see [26] and references therein for a review in such developments). In particular, the calculation of 1-loop amplitudes with external massive quarks are crucial to describe heavy quark hadroproduction and constant theoretical developments have led to numerical or semi-analytical procedures that provide results with improved precision. These advances have allowed for a continuous flow of different mathematical methods enabling scalar and tensor loop integral evaluation with massive lines [26].

In this article, in addition to constructing a WFGTI conserving longitudinal Compton scattering vertex, we also present its complete 1-loop calculation for off-shell external momenta and for arbitrary gauge and dimensions for massive SQED. This involves 2-, 3- and 4-point scalar and tensor integrals up to 2 indices <sup>1</sup>. The box integrals involved have at most three massive propagators. We proceed by writing down the contributing diagrams and calculating them in terms of scalar and tensor integrals which we list in the appendices. Due to exchange symmetries, only eleven of the twenty eight diagrams are independent. All the calculations have been carried out using FORM or/and Mathematica 6.0.

We have organized the article as follows: In section II, we start out by proposing a convenient basis to decompose the Compton scattering vertex into its longitudinal and transverse parts. Moreover, we construct the longitudinal vertex non perturbatively. It exhausts 5 of the 10 basis tensors. In Section II, after setting the notation and introducing the Feynman rules, we present all the 1-loop topologies for the Compton scattering and write them in terms of tensor and scalar integrals to be evaluated. The reduction of the tensor integrals in terms of scalar ones and their evaluation has been done in the appendices. Subtraction of the longitudinal part earlier constructed yields the undetermined transverse vertex to the one loop order. This should provide us with a guide towards its possible non perturbative extensions. In section IV, we present our conclusions. The appendices have been dedicated to the calculation of the necessary integrals and presenting the 1-loop results for the full 4-point vertex.

#### II. WGTI CONSERVING LONGITUDINAL VERTEX

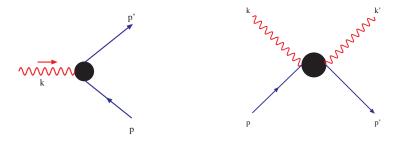
The WFGTI which relates 3- and 4-point Green functions is:

$$k'^{\mu}\Gamma_{\nu\mu}(p',k';p,k) = \Gamma_{\nu}(p+k,p) - \Gamma_{\nu}(p',p'-k) ,$$
  

$$k^{\mu}\Gamma_{\nu\mu}(p',k';p,k) = \Gamma_{\nu}(p',p'+k') - \Gamma_{\nu}(p-k',p) ,$$
(2)

where the vertices involved are full, as shown in the diagram below:

<sup>&</sup>lt;sup>1</sup> Tensor integrals involving three Lorentz indices also arise but the two diagrams involving them cancel each other out because of their charge conjugation symmetry.



The first argument of the 3-point vertex throughout represents the momentum of the incoming fermion and the second that of the outgoing one. The WFGTI identities allow us to write "longitudinal parts" of the 4-point vertex in term of the 3-point vertex. These parts are defined as the ones which vanish on carrying out any of the contractions described in Eq. (2). The remaining "transverse parts" will have to be calculated by the brute force of perturbation theory. We conveniently define

$$Q_{\mu} = k'_{\mu}(p+p') \cdot k - k \cdot k'(p+p')_{\mu} ,$$

$$Q'_{\nu} = k_{\nu}(p+p') \cdot k' - k \cdot k'(p+p')_{\nu} ,$$

$$R_{\mu} = k_{\mu}k' \cdot k - k^{2}k'_{\mu} ,$$

$$R'_{\nu} = k'_{\nu}k' \cdot k - k'^{2}k_{\nu} .$$
(3)

Note that this definition is suitable to separate out the transverse part of the vertex from the longitudinal one. Thus the complete 4-point vertex can be written in its most general form as:

$$\Gamma_{\mu\nu} = \Gamma_{\mu\nu}^{L} + \Gamma_{\mu\nu}^{T} = Ag_{\mu\nu} + B_{11}(k \cdot k'g_{\nu\mu} - k_{\nu}k'_{\mu}) + B_{12}Q'_{\nu}k'_{\mu} + B_{13}R'_{\nu}k'_{\mu} + B_{21}k_{\nu}Q_{\mu} + B_{22}Q'_{\nu}Q_{\mu} + B_{23}R'_{\nu}Q_{\mu} + B_{31}k_{\nu}R_{\mu} + B_{32}Q'_{\nu}R_{\mu} + B_{33}R'_{\nu}R_{\mu} ,$$

$$(4)$$

where the coefficients A,  $B_{12}$ ,  $B_{21}$ ,  $B_{31}$  and  $B_{13}$  are determined by WFGT identity to all orders in perturbation theory. Therefore,

$$\Gamma^{L}_{\mu\nu} = Ag_{\mu\nu} + B_{12}Q'_{\nu}k'_{\mu} + B_{13}R'_{\nu}k'_{\mu} + B_{21}k_{\nu}Q_{\mu} + B_{31}k_{\nu}R_{\mu} , \qquad (5)$$

$$\Gamma_{\mu\nu}^{T} = B_{11}(k \cdot k' g_{\nu\mu} - k_{\nu} k'_{\mu}) + B_{22} Q'_{\nu} Q_{\mu} + B_{23} R'_{\nu} Q_{\mu} + B_{32} Q'_{\nu} R_{\mu} + B_{33} R'_{\nu} R_{\mu} . \tag{6}$$

The coefficients which constitute the longitudinal part of the vertex are

$$A = -\frac{1}{k \cdot k'} \left\{ \Delta^{-1}(p+k) - \Delta^{-1}(p) + \Delta^{-1}(p'-k) - \Delta^{-1}(p') \right\}$$

$$B_{21} = -\frac{1}{k \cdot k'} \left\{ \frac{\Delta^{-1}(p+k) - \Delta^{-1}(p)}{(p+k)^{2} - p^{2}} - \frac{\Delta^{-1}(p') - \Delta^{-1}(p'-k)}{p'^{2} - (p'-k)^{2}} - k^{2} \left[ \Gamma_{T}(p+k,p) - \Gamma_{T}(p',p'-k) \right] \right\}$$

$$B_{12} = -\frac{1}{k \cdot k'} \left\{ \frac{\Delta^{-1}(p'-k') - \Delta^{-1}(p')}{(p'-k')^{2} - p'^{2}} - \frac{\Delta^{-1}(p) - \Delta^{-1}(p+k')}{p^{2} - (p+k')^{2}} - k'^{2} \left[ \Gamma_{T}(p'-k',p') - \Gamma_{T}(p,p+k') \right] \right\}$$

$$B_{31} = \frac{1}{(k \cdot k')^{2}} \left\{ \left[ (p+k)^{2} - p^{2} \right] \Gamma_{T}(p+k,p) + \left[ (p'-k)^{2} - p'^{2} \right] \Gamma_{T}(p',p'-k') \right\}$$

$$B_{13} = \frac{1}{(k \cdot k')^{2}} \left\{ \left[ (p'-k')^{2} - p'^{2} \right] \Gamma_{T}(p'-k',p') + \left[ (p+k')^{2} - p^{2} \right] \Gamma_{T}(p,p+k') \right\},$$

$$(7)$$

where  $\Gamma_T$  is the transverse part of the 3-point scalar-photon vertex defined as

$$\Gamma_{\mu}(p+k,p) = (2p+k)_{\mu} \frac{\Delta^{-1}(p+k) - \Delta^{-1}(p)}{(p+k)^2 - p^2} + 2(k_{\mu}p \cdot k - k^2p_{\mu})\Gamma_T(p+k,p) . \tag{8}$$

Note that in all those calculations, where only the longitudinal projection of the true vertex is involved, Eqs. (7) constitute the exact non perturbative result. However,  $B_{11}$ ,  $B_{22}$ ,  $B_{23}$ ,  $B_{32}$  and  $B_{33}$  remain undetermined and need to be calculated perturbatively order by order.

In the next section, we calculate the transverse vertex to one loop order in perturbation theory. This requires the calculation of the full vertex to that order.

#### III. COMPTON SCATTERING VERTEX

#### A. Preliminaries

In this sub-section, we shall set the notation and define all the scalar and tensor integrals to be evaluated. We define the bare quantities in the usual form: the scalar propagator  $\Delta(p) = 1/(p^2 - m^2)$ , the photon propagator  $\Delta_{\mu\nu}^0 = -\left[g_{\mu\nu}p^2 - (1-\xi)p_{\mu}p_{\nu}\right]/p^4$ , the 3-point vertex  $\Gamma_{\mu}^0 = (k+p)_{\mu}$  and the 4-point double photon vertex  $e^2\Gamma_{\mu\nu}^0 = e^2g_{\mu\nu}$ , where  $\xi$  is the general covariant gauge parameter (such that  $\xi = 0$  corresponds to Landau gauge) and e is the usual QED coupling constant. Following is the list of 1-, 2-, 3- and 4-point scalar integrals that we need to evaluate:

$$T = \int \frac{d^D w}{[w^2 - m^2]} \,, \tag{9}$$

$$K(k) = \int \frac{d^D w}{w^2 [(k-w)^2 - m^2]}, \qquad (10)$$

$$\widetilde{K}(k) = \int \frac{d^D w}{[w^2 - m^2][(k - w)^2 - m^2]},$$
(11)

$$L(k) = \int \frac{d^D w}{w^4 [(k-w)^2 - m^2]}, \qquad (12)$$

$$I(k,p) = \int \frac{d^D w}{w^2[(k-w)^2 - m^2][(p-w)^2 - m^2]},$$
(13)

$$\widetilde{I}(k,p) = \int \frac{d^D w}{[w^2 - m^2][(k-w)^2 - m^2][(p-w)^2 - m^2]}$$
(14)

$$J(k,p) = \int \frac{d^D w}{w^4 [(k-w)^2 - m^2][(p-w)^2 - m^2]}, \qquad (15)$$

$$U(k, p, q) = \int \frac{d^D w}{w^2 [(k - w)^2 - m^2][(p - w)^2 - m^2][(q - w)^2 - m^2]},$$
 (16)

$$V(k,p,q) = \int \frac{d^D w}{w^4 [(k-w)^2 - m^2][(p-w)^2 - m^2][(q-w)^2 - m^2]}.$$
 (17)

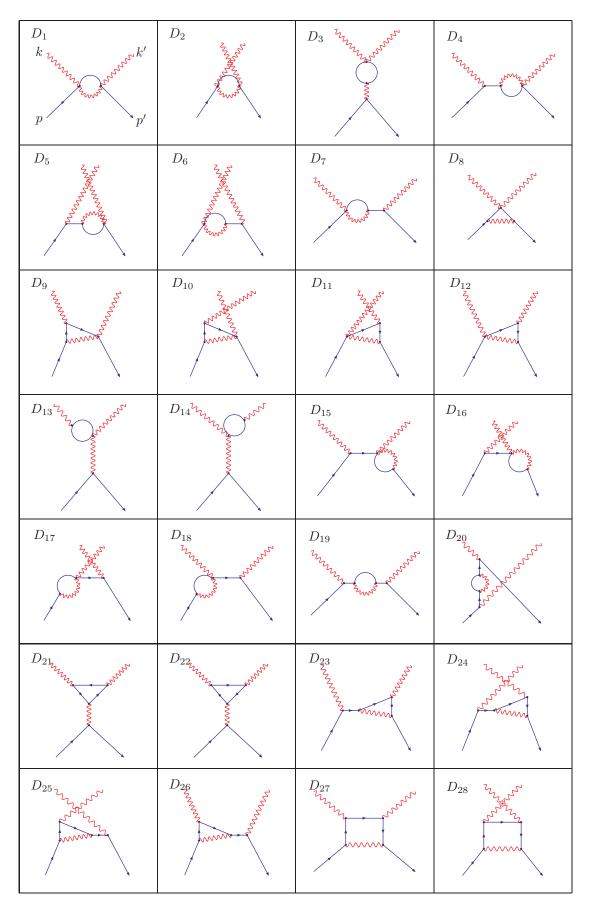
We also meet tensor integrals up to two indices in such a way that the numerator is  $w^{\mu}$  or  $w^{\mu}w^{\nu}$  with the same denominators as in Eqs. (9-17). Therefore, we shall employ the same notation as above for these tensor integrals with the only difference that we shall add an appropriate tensor superscript (or subscript) to the representing symbol. Thus, for example,

$$\widetilde{I}^{\mu\nu}(k,p) = \int \frac{w^{\mu}w^{\nu} d^{D}w}{[w^{2} - m^{2}][(k-w)^{2} - m^{2}][(p-w)^{2} - m^{2}]}.$$
(18)

This set of integrals appear in all the possible 1-loop Feynman graphs contributing to the Compton scattering process, as we shall see next. Results of all these integrals are given in detail in the appendices.

Some of these integrals diverge in 4-dimensions and need to be regularized and renormalized. We do not carry out renormalization in this article but it is worth mentioning that the scalar integrals T, K(k), L(k), J(k,p) and V(k,p) are divergent, whereas, I(k,p),  $\widetilde{I}(k,p)$  and U(k,p,q) are convergent. As the tensor integrals can be decomposed in terms of scalar integrals, it is easy to deduce their convergence properties.

# B. Compton scattering diagrams



In this section, we express all these diagrams in terms of the scalar and multi-indexed integrals defined in the previous section. Note that we use the notation q = k + p and q' = k' + p throughout:

$$\Gamma_{D_1}^{\mu\nu}(q) = -\frac{2ie^2}{(2\pi)^D} \left\{ -g^{\mu\nu}K(q) + (1-\xi)L^{\mu\nu}(q) \right\}, \tag{19}$$

$$\Gamma_{D_4}^{\mu\nu}(p,q) = \frac{ie^2}{(2\pi)^D} \left\{ -2\frac{(p^{\mu} + q^{\mu})q^{\nu}}{(q^2 - m^2)} K(q) + \frac{(p^{\mu} + q^{\mu})}{(q^2 - m^2)} K^{\nu}(q) + (1 - \xi)(p^{\mu} + q^{\mu})L^{\nu}(q) \right\}, \tag{20}$$

$$\Gamma_{D_8}^{\mu\nu} = \frac{-ie^2}{(2\pi)^D} g^{\mu\nu} \bigg\{ \widetilde{K}(p-p') - 2(p+p')^{\mu} I_{\mu}(p,p') + 4p \cdot p' I(p,p') \bigg\}$$

$$+(\xi-1)[\widetilde{K}(p-p')-2(p+p')^{\mu}I_{\mu}(p,p')+4p'^{\mu}p^{\nu}J_{\mu\nu}(p,p')]\right\},$$
(21)

$$\Gamma_{D_9}^{\mu\nu}(p,q) = \frac{-ie^2}{(2\pi)^D} \left\{ 2(p+q)^{\mu}p^{\nu}I(p,q) - (p+q)^{\mu}I^{\nu}(p,q) - 4p^{\nu}I^{\mu}(p,q) + 2I^{\mu\nu}(p,q) - (1-\xi)\left[(p^2 - m^2)(p+q)^{\mu}J^{\nu}(p,q) - 2(p^2 - m^2)J^{\mu\nu}(p,q) + 2L^{\mu\nu}(q) - (p+q)^{\mu}L^{\nu}(q)\right] \right\},$$
(22)

$$\Gamma_{D_{15}}^{\mu\nu}(p,q,p') = \frac{ie^2}{(2\pi)^D} \frac{(p+q)^{\mu}}{(q^2-m^2)} \left\{ -2p'^{\nu}K(p') + K^{\nu}(p') + (1-\xi)(p'^2-m^2)L^{\nu}(p') \right\}, \tag{23}$$

$$\Gamma_{D_{19}}^{\mu\nu} = \frac{ie^2}{2(2\pi)^D}(p+q)^{\mu}(p'+q)^{\nu} \left\{ \left( \frac{2}{q^2-m^2} + \frac{4m^2}{(q^2-m^2)^2} \right) K(q) - \frac{T}{(q^2-m^2)^2} - \frac{(1-\xi)}{q^2-m^2} L(q) \right\}, \quad (24)$$

$$\Gamma_{D_{20}}^{\mu\nu} = \frac{-ie^2}{2(2\pi)^D} \frac{(p'+q')^{\mu}(p+q')^{\nu}}{(q'^2-m^2)^2} \left\{ -4q'^2K(q') - T + 4q'_{\alpha}K^{\alpha}(q') + (1-\xi)\left[T - 4q'_{\alpha}K^{\alpha}(q') + (1-\xi)\left[T - 4q'_{\alpha}K^{\alpha}(q')\right]\right] \right\},$$

$$\left. +4q'_{\alpha}q'_{\beta}L^{\alpha\beta}(q')\right] \right\},$$
(25)

$$\Gamma_{D_{23}}^{\mu\nu}(p,q,p') = \frac{ie^2}{2(2\pi)^D} \left\{ (p+q)^{\mu}(p'+q)^{\nu} \left[ \left( \frac{4p' \cdot q}{q^2 - m^2} + \frac{m^2 - p'^2}{q^2 - m^2} - 1 \right) I(p',q) - \frac{\widetilde{K}(p'-q)}{q^2 - m^2} + \frac{K(p')}{q^2 - m^2} \right. \\ \left. + \frac{K(q)}{q^2 - m^2} \right] - 2(p+q)^{\mu} \left[ \left( \frac{4p' \cdot q}{q^2 - m^2} + \frac{m^2 - p'^2}{q^2 - m^2} - 1 \right) I^{\nu}(p',q) - \frac{p'^{\nu} + q^{\nu}}{2(q^2 - m^2)} \widetilde{K}(p'-q) \right. \\ \left. + \frac{K^{\nu}(p')}{q^2 - m^2} + \frac{K^{\nu}(q)}{q^2 - m^2} \right] - (1 - \xi) \left[ (p+q)^{\mu}(p'+q)^{\nu} \left[ (p'^2 - m^2)J(p',q) - L(q) \right. \\ \left. - \frac{p'^2 - m^2}{q^2 - m^2} L(p') \right] - 2(p+q)^{\mu} \left[ (p'^2 - m^2)J^{\nu}(p',q) - L^{\nu}(q) - \frac{p'^2 - m^2}{q^2 - m^2} L^{\nu}(p') \right] \right] \right\}, \tag{26}$$

$$\begin{split} \Gamma^{\mu\nu}_{D_{27}}(k,p,q,p') &= \frac{-ie^2}{2(2\pi)^D} \Big\{ (k+2p)^\mu (k+p+p')^\nu [(-2m^2+(p'-p)^2-2p\cdot p')U(p,p',q) + \widetilde{I}(q-p,p'-p) \\ &- I(p,q) - I(p',q)] - 2(k+p+p')^\nu [(-2m^2+(p'-p)^2-2p\cdot p')U^\mu (p,p',q) \\ &+ \widetilde{I}^\mu (q-p,p'-p) - I^\mu (p,q) - I^\mu (p',q)] - (2k+4p)^\mu [(-2m^2+(p'-p)^2-2p\cdot p')U^\nu (p,p',q) \\ &+ \widetilde{I}^\nu (q-p,p'-p) - I^\nu (p,q) - I^\nu (p',q)] + 4 [(-2m^2+(p'-p)^2-2p\cdot p')U^{\mu\nu} (p,p',q) \\ &+ \widetilde{I}^{\mu\nu} (q-p,p'-p) - I^{\mu\nu} (p,q) - I^{\mu\nu} (p',q)] + (1-\xi) \big[ (k+2p)^\mu (k+p+p')^\nu \big[ (-m^2(p^2+p'^2-m^2) + p^2p'^2)V(p,p',q) + (p^2+p'^2-2m^2)U(p,p',q) + \widetilde{I}(q-p,p'-p) + (m^2-p^2)J(p,q) \\ &+ (m^2-p'^2)J(p',q) - I(p,q) - I(p',q) + L(q) \big] - 2(k+p+p')^\nu \big[ (-m^2(p^2+p'^2-m^2) + p^2p'^2)V^\mu (p,p',q) + (p^2+p'^2-2m^2)U^\mu (p,p',q) + \widetilde{I}^\mu (q-p,p'-p) + (m^2-p^2)J^\mu (p,q) \\ &+ (m^2-p'^2)J^\mu (p',q) - I^\mu (p,q) - I^\mu (p',q) + L^\mu (q) \big] - (2k+4p)^\mu \big[ (-m^2(p^2+p'^2-m^2) + p^2p'^2)V^\nu (p,p',q) + (p^2+p'^2-2m^2)U^\nu (p,p',q) + \widetilde{I}^\nu (q-p,p'-p) + (m^2-p^2)J^\nu (p,q) \\ &+ (m^2-p'^2)J^\nu (p',q) - I^\nu (p,q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p,q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p,q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p,q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + L^\nu (q) \big] + 4 \big[ (-m^2(p^2+p'^2-m^2) + (m^2-p'^2)J^\nu (p',q) - I^\nu (p',q) - I^\nu (p',q) + I^\nu (p',q) + I^\nu (q',q) + I^\nu (q',q) + I^\nu (q',q) + I^\nu ($$

$$+p^{2}p'^{2})V^{\mu\nu}(p,p',q) + (p^{2} + p'^{2} - 2m^{2})U^{\mu\nu}(p,p',q) + \widetilde{I}^{\mu\nu}(q-p,p'-p) + (m^{2} - p^{2})J^{\mu\nu}(p,q) + (m^{2} - p'^{2})J^{\mu\nu}(p',q) - I^{\mu\nu}(p,q) - I^{\mu\nu}(p',q) + L^{\mu\nu}(q)] \}.$$
(27)

The rest of the diagrams can be obtained as follows:

$$\Gamma_{D_2}^{\mu\nu} = \Gamma_{D_1}^{\mu\nu}(q'),$$
(28)

$$\Gamma_{D_5}^{\mu\nu} = \Gamma_{D_4}^{\mu\nu}(p, q'), \qquad \Gamma_{D_6}^{\mu\nu} = \Gamma_{D_4}^{\mu\nu}(p', q'), \qquad \Gamma_{D_7}^{\mu\nu} = \Gamma_{D_4}^{\mu\nu}(p', q), \tag{29}$$

$$\Gamma_{D_{10}}^{\mu\nu} = \Gamma_{D_9}^{\mu\nu}(p, q'), \qquad \Gamma_{D_{11}}^{\mu\nu} = \Gamma_{D_9}^{\mu\nu}(p', q'), \qquad \Gamma_{D_{12}}^{\mu\nu} = \Gamma_{D_9}^{\mu\nu}(p', q),$$
 (30)

$$\Gamma_{D_{16}}^{\mu\nu} = \Gamma_{D_{15}}^{\mu\nu}(p, q', p'), \quad \Gamma_{D_{17}}^{\mu\nu} = \Gamma_{D_{15}}^{\mu\nu}(p', q', p), \quad \Gamma_{D_{18}}^{\mu\nu} = \Gamma_{D_{15}}^{\mu\nu}(p', q, p), \tag{31}$$

$$\Gamma_{D_{24}}^{\mu\nu} = \Gamma_{D_{23}}^{\mu\nu}(p, q', p'), \quad \Gamma_{D_{25}}^{\mu\nu} = \Gamma_{D_{23}}^{\mu\nu}(p', q', p), \quad \Gamma_{D_{26}}^{\mu\nu} = \Gamma_{D_{23}}^{\mu\nu}(p', q, p), \tag{32}$$

$$\Gamma_{D_{28}}^{\mu\nu} = \Gamma_{D_{27}}^{\mu\nu}(k', p, q', p'), \tag{33}$$

while the contributions  $\Gamma^{\mu\nu}_{D_3}$ ,  $\Gamma^{\mu\nu}_{D_{13}}$ ,  $\Gamma^{\mu\nu}_{D_{14}}$ ,  $\Gamma^{\mu\nu}_{D_{21}} + \Gamma^{\mu\nu}_{D_{22}}$  all vanish. Evaluation of the integrals completes the calculation of the 1-loop Compton scattering amplitude for off-shell external legs in arbitrary gauge and dimensions in SQED. Results for all the scalar and multi-indexed tensor integrals have been presented in the appendices. Through their most general decompositions in terms of the available vectors, the tensor integrals are converted into a series of scalar ones for their subsequent evaluation. Most of these results are new. We verify them to reduce to known expressions for simpler cases whenever possible. Moreover, as another confirmatory check, we verify the properties of the scalar pieces which reflect the symmetries of the original integrals.

Once we know the full vertex at the one loop order, we can subtract from it the longitudinal part of the previous section to extract the unknown transverse coefficients  $B_{11}$ ,  $B_{22}$ ,  $B_{23}$ ,  $B_{32}$  and  $B_{33}$  to 1-loop in perturbation theory. Although straightforward in principle, it is rather involved in practice. The outline is as follows. The 4-point Compton scattering vertex can be expanded in a natural basis as:

$$\Gamma_{\mu\nu} = C_0 q_{\mu\nu} + C_1 k_{\mu}k_{\nu} + C_2 k_{\mu}p_{\nu} + C_3 p_{\mu}k_{\nu} + C_4 k_{\mu}p'_{\nu} + C_5 p'_{\nu}k_{\nu} + C_6 p_{\mu}p'_{\nu} + C_7 p'_{\nu}p_{\nu} + C_8 p'_{\nu}p'_{\nu} + C_9 p_{\mu}p_{\nu}, (34)$$

where the coefficients  $C_i$  can be identified from the expressions (19-27). Once the  $C_i$  are known, then comparing Eq. (4) and Eq. (34) we can write the unknown transverse pieces in Eq. (4) at the 1-loop level as follows:

$$B_{11} = -\frac{k^2}{k \cdot k'} C_1 + \frac{k^2(k \cdot p + p^2 - p \cdot p')}{(k \cdot k')^2} C_2 - \frac{k \cdot p}{k \cdot k'} C_3 + \frac{k^2(-p'^2 + p \cdot p' + k \cdot p')}{(k \cdot k')^2} C_4$$

$$-\frac{k \cdot p'}{k \cdot k'} C_5 + \frac{k \cdot p(-p'^2 + p \cdot p' + k \cdot p')}{(k \cdot k')^2} C_6 + \frac{k \cdot p'(k \cdot p + p^2 - p \cdot p')}{(k \cdot k')^2} C_7$$

$$+\frac{k \cdot p'(-p'^2 + p \cdot p' + k \cdot p')}{(k \cdot k')^2} C_8 + \frac{k \cdot p(k \cdot p + p^2 - p \cdot p')}{(k \cdot k')^2} C_9 ,$$

$$B_{22} = \frac{1}{4} \frac{C_6 + C_7 + C_8 + C_9}{(k \cdot p')(-2k \cdot p + k \cdot p') + (k \cdot p)^2 + k^2(k^2 + 2k \cdot p - 2k \cdot p')} ,$$

$$B_{23} = \frac{1}{4} \frac{C_6 - C_7 + C_8 - C_9}{(k \cdot p')(-2k \cdot p + k \cdot p') + (k \cdot p)^2 + k^2(k^2 + 2k \cdot p - 2k \cdot p')} ,$$

$$B_{32} = \frac{1}{4} \frac{2 C_2 + 2 C_4 - C_6 + C_7 + C_8 - C_9}{(k \cdot p')(-2k \cdot p + k \cdot p') + (k \cdot p)^2 + k^2(k^2 + 2k \cdot p - 2k \cdot p')} ,$$

$$B_{33} = -\frac{1}{4} \frac{2 C_2 - 2 C_4 + C_6 + C_7 - C_8 - C_9}{(k \cdot p')(-2k \cdot p + k \cdot p') + (k \cdot p)^2 + k^2(k^2 + 2k \cdot p - 2k \cdot p')} .$$

$$(35)$$

Thus the decomposition of the Compton scattering vertex is completely defined by Eqs. (3-8) and Eq. (35). The longitudinal part has been determined non perturbatively, whereas the transverse part has been evaluated to 1-loop order. The perturbative transverse pieces can guide us to their possible non perturbative structures which should reduce to this expansion in the weak coupling regime.

#### IV. CONCLUSIONS

In this article, we provide a suitable basis to decompose the full Compton scattering vertex in SQED into its longitudinal and transverse (to one or both external photons) parts. We then employ the WFGTI which relates 3-point vertices to the 4-point ones in order to determine the longitudinal component of the Compton scattering vertex in an exact non perturbative fashion, written in terms of lower point functions. Recall that the WGTI-conserving model building for the three point interactions provides us with a reliable truncation scheme for the SDEs at the level of two-point functions. The work presented in this article provides an opportunity to lead us to the next order of approximation. One can now truncate the tower of SDE at the level of the 4-point function, again with a WFGTI conserving ansatz and consistently solve the coupled set of SDEs for the 2- and the 3-point functions in order to arrive at their more accurate solutions. A natural next step is to probe the non perturbative and vitally important structures of 4-point vertices in QCD through the generalized Slavnov-Taylor identities.

As the name suggests, the transverse piece remains undetermined by the WFGTI. However, with the help of our complete 1-loop evaluation of this vertex, the unknown transverse pieces can be extracted to that order. It should serve as a guide for its possible non perturbative extensions. Any such attempt should reproduce this perturbative result in the weak coupling regime.

There are further advantages of calculating Compton-scattering for massive electrons and off-shell external legs in arbitrary gauge and dimensions: (i) As the external legs are off-shell, one can study various cases of interest by putting the desired particles on shell. On the other hand, it can also serve as the internal part of more elaborate Feynman diagrams. (ii) Arbitrary gauge helps in checking the gauge invariance of the related physical observables and the gauge covariance properties of the Green functions. (iii) Substituting  $D=4-2\epsilon$  and expanding in powers of  $\epsilon$ , we can study the 4-dimensional case. Interest has also been shown in lower dimensional SQED which can be simply projected out by an appropriate substitution of D. (iv) Explicit results of most of the integrals are new and can also be used in other calculations. These are the same integrals which are also encountered in spinor QED and QCD. Therefore, this study can provide a guideline for a similar calculation of the ghost-ghost-quark-quark, 4-gluon and 4-quark vertices in QCD, whose importance has been advocated in the introductory section.

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### **APPENDIX I: Master Integrals**

In this appendix we summarize the results for various integrals that have appeared throughout the calculation. Although some of the integrals are basic and already known, we tabulate them all for the sake of completeness. All through the appendix, we use the simplifying convention  $X_0 = (2/i\pi^2)X$  for all scalar integrals with or without tilde.

#### A. Scalar Integrals

## The 1- and 2-Point Integrals:

We start from the list of all the scalar integrals up to 2-point integrals:

$$T = -i\pi^{D/2} (m^2)^{D/2-1} \Gamma\left(1 - \frac{D}{2}\right), \tag{36}$$

$$K(q) = -i\pi^{D/2}(m^2)^{D/2-2}\Gamma\left(1 - \frac{D}{2}\right) {}_{2}F_{1}\left(2 - \frac{D}{2}, 1; \frac{D}{2}; \frac{q^2}{m^2}\right), \tag{37}$$

$$\widetilde{K}(q) = i\pi^{D/2} (m^2)^{D/2-2} \Gamma\left(2 - \frac{D}{2}\right) {}_{2}F_{1}\left(1, 2 - \frac{D}{2}; \frac{3}{2}; \frac{q^2}{4m^2}\right) , \tag{38}$$

$$L(q) = -i\pi^{D/2}(m^2)^{D/2-3}\Gamma\left(1-\frac{D}{2}\right){}_{2}F_{1}\left(3-\frac{D}{2},2;\frac{D}{2};\frac{q^2}{m^2}\right). \tag{39}$$

Some of these scalar integrals can be compared with those reported elsewhere.

- One can check that K(q) is the same as  $J(\alpha, \beta, 0, m)$  in Eq. (10) of reference [11] for  $\alpha = \beta = 1$ .
- Similarly L(q) is the same as  $J(\alpha, \beta, 0, m)$  (as above) for  $\alpha = 2$  and  $\beta = 1$ . Moreover, using the recurrence relation (A.17) of [28], one can see that L(q) is not independent. It can be written in terms of T and K(q) as follows:

$$L(q) = -\frac{1}{(q^2 - m^2)^2} \left[ (D - 3) \left( q^2 + m^2 \right) K(q) - (D - 2)T \right]. \tag{40}$$

• Eq. (17) of [11] in the case  $\nu_1 = \nu_2 = 1$ , p = q reproduces  $\tilde{K}(q)$ . It can also be obtained from  $J(1,1;m_1,m_2)$ , Eqs. (A.1, A.7) of [28] for  $m_1 = m_2$  and  $x = y = m^2/q^2$  and after simplification.

## The 3-Point Integrals:

We now come to the 3-point function of two variables k and p:

$$I(k,p) = (m^2)^{D/2-3} \frac{\Gamma(3-\frac{D}{2})}{1-\frac{D}{2}} \Phi_2 \left[ \begin{array}{cc} 3-\frac{D}{2}, 1, 1, 1; \frac{D}{2}-1 \\ \frac{D}{2}; 2 \end{array} \middle| \frac{k^2}{m^2}, \frac{p^2}{m^2}, \frac{(k-p)^2}{m^2} \right], \tag{41}$$

$$\widetilde{I}(k,p) = -(m^2)^{D/2-3} \frac{\Gamma(3-\frac{D}{2})}{2} \Phi_3 \left[ \begin{array}{cc} 3 - \frac{D}{2}, 1, 1, 1 \\ 3 \end{array} \middle| \frac{k^2}{m^2}, \frac{(k-p)^2}{m^2}, \frac{p^2}{m^2} \right], \tag{42}$$

$$J(k,p) = \frac{1}{2\chi} \left\{ -2(D-4)(k-p)^2 \left[ m^2 + (kp) \right] I(k,p) + 2(D-3)(k-p)^2 \tilde{K}(k-p) - (D-2) \frac{(k-p)^2}{m^2} T - \frac{(p^2 - m^2)(k-p)^2 - 2(p^2 - k^2)m^2}{(p^2 - m^2)^2} \left[ 2(D-3)p^2 K(p) - (D-2) \frac{p^2 + m^2}{2m^2} T \right] - \frac{(k^2 - m^2)(k-p)^2 + 2(p^2 - k^2)m^2}{(k^2 - m^2)^2} \left[ 2(D-3)k^2 K(k) - (D-2) \frac{k^2 + m^2}{2m^2} T \right] \right\},$$

$$(43)$$

where  $\chi = m^2(k^2 - p^2)^2 + (m^2 - k^2)(m^2 - p^2)(k - p)^2$ . Note that the the integrals I(k, p) and J(k, p) are  $J_2(1, 1, 1)$  and  $J_2(1, 1, 2)$  of [6]. Generalized Lauricella function  $\Phi_2$  arises for general powers of three propagtors  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  (with the propagator corresponding to  $\nu_1$  being massless, see [11, 16, 17] and references therein, for definition, properties and symmetry relations):

$$\Phi_{2} \left[ \begin{array}{c} \nu_{123} - \frac{D}{2}, \nu_{1}, \nu_{2}, \nu_{3}; \frac{D}{2} - \nu_{1} \\ \frac{D}{2}; \nu_{23} \end{array} \middle| x_{1}, x_{2}, x_{3} \right] = \sum_{n,l,j=0}^{\infty} \frac{x_{1}^{n} x_{2}^{l} x_{3}^{j}}{n! \ l! \ j!} \times \frac{\left(\nu_{123} - \frac{D}{2}; n + l + j\right) \left(\nu_{1}; n + l\right) \left(\nu_{2}; n + j\right) \left(\nu_{3}; l + j\right) \left(\frac{D}{2} - \nu_{1}; j\right)}{\left(\frac{D}{2}; n + l + j\right) \left(\nu_{23}; n + l + 2j\right)} , (44)$$

where the Pochhammer symbol  $(z; n) = \Gamma(z+n)/\Gamma(z)$  and  $\nu_{i_1,i_2,\dots,i_j} = \nu_{i_1} + \nu_{i_2} + \dots + \nu_{i_j}$ . Similarly, the generalized Lauricella function  $\Phi_3$ <sup>2</sup> arises for general powers of three massive propagtors:  $\nu_1$ ,  $\nu_2$  and  $\nu_3$ ,

$$\Phi_{3}\left[\begin{array}{c|c}\nu_{123} - \frac{D}{2}, \nu_{1}, \nu_{2}, \nu_{3}\\\nu_{123}\end{array} \middle| x_{1}, x_{2}, x_{3}\right] = \sum_{n,l,j=0}^{\infty} \frac{x_{1}^{n} x_{2}^{l} x_{3}^{j}}{n! \ l! \ j!} \frac{\left(\nu_{123} - \frac{D}{2}; n+l+j\right)\left(\nu_{1}; l+j\right)\left(\nu_{2}; n+j\right)\left(\nu_{3}; n+l\right)}{\left(\nu_{123}; n+l+2j\right)} . (45)$$

We finally go on to the 4-point integrals.

<sup>&</sup>lt;sup>2</sup> Note that the generalized Lauricella function was first introduced in [29]. Some special cases, including those denoted as  $\Phi_2$  and  $\Phi_3$ , were employed in [11]. Further examples can be found in [16, 17].

## The 4-Point Integrals:

Genuine 4-point integrals are given by Eq. (16) and Eq. (17). We evaluate them using the Mellin-Barnes technique developed in  $[11, 16, 17]^3$ :

$$U(p,q,p') = i\pi^{D/2}(m^2)^{D/2-4} \frac{\Gamma(\frac{D}{2}-1)\Gamma(4-\frac{D}{2})}{2\Gamma(\frac{D}{2})} \times$$

$$\mathcal{Y}_3 \left[ \begin{array}{ccc} 4 - \frac{D}{2}, 1, 1, 1, 1, \frac{D}{2} - 1 & \frac{p^2}{m^2}, \frac{q^2}{m^2}, \frac{p'^2}{m^2}, \frac{(q-p)^2}{m^2}, \frac{(p-p')^2}{m^2}, \frac{(p-p')^2}{m^2} \end{array} \right],$$

$$V(p,q,p') = -i\pi^{D/2}(m^2)^{D/2-5} \frac{\Gamma(\frac{D}{2}-2)\Gamma(5-\frac{D}{2})}{6\Gamma(\frac{D}{2})} \times$$

$$\mathcal{Y}_3 \left[ \begin{array}{cccc} 5 - \frac{D}{2}, 1, 1, 1, 2, \frac{D}{2} - 2 & \frac{p^2}{m^2}, \frac{q^2}{m^2}, \frac{p'^2}{m^2}, \frac{(q-p)^2}{m^2}, \frac{(q-p')^2}{m^2}, \frac{(p-p')^2}{m^2} \end{array} \right].$$

$$(46)$$

The scalar integrals U(p, q, p') and V(p, q, p'), given by Eqs. (46, 47) are genuine 4-point functions with three massive propagators, the most non-trivial integrals calculated in this paper. The function  $\mathcal{Y}_3$  is given by

$$\mathcal{Y}_{3}\left[\begin{array}{c}\nu_{1234} - \frac{D}{2}, \nu_{1}, \nu_{2}, \nu_{3}, \nu_{4}; \frac{D}{2} - \nu_{4} \\ \frac{D}{2}, \nu_{123}\end{array}\right| x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}\right] = \sum_{n, l, r, w, k, j = 0}^{\infty} \frac{x_{1}^{n} x_{2}^{l} x_{3}^{r} x_{4}^{w} x_{5}^{k} x_{6}^{j}}{n! \ l! \ r! \ w! \ k! \ j!} \times \frac{\left(\nu_{1234} - \frac{D}{2}; n + l + r + w + k + j\right)\left(\nu_{1}; r + k + j\right)\left(\nu_{2}; n + l + r\right)\left(\nu_{3}; n + w + j\right)\left(\nu_{4}; l + w + k\right)\left(\frac{D}{2} - \nu_{4}; n + r + j\right)}{\left(\frac{D}{2}; n + l + r + w + k + j\right)\left(\nu_{123}; 2n + l + 2r + w + k + 2j\right)}. (48)$$

This completes the calculation of the scalar integrals.

#### B. Vector Integrals

The more straightforward vector integrals which are functions of just one momentum variable are:

$$K^{\mu}(q) = \frac{q^{\mu}}{2q^2} \left[ T - (m^2 - q^2)K(q) \right] , \qquad (49)$$

$$\widetilde{K}^{\mu}(q) = \frac{q^{\mu}}{2}\widetilde{K}(q) , \qquad (50)$$

$$L^{\mu}(q) = \frac{q^{\mu}}{2q^2} \left[ (q^2 - m^2) L(q) + K(q) \right] . \tag{51}$$

Relatively more involved vector integrals are functions of two momentum variables which we take up now.

## The $I^{\mu}(k,p)$ Integral:

The most general description of  $I^{\mu}(k,p)$  is:

$$I_{\mu}(k,p) = \frac{i\pi^2}{2} [k_{\mu} I_A(k,p) + p_{\mu} I_B(k,p)], \qquad (52)$$

where

$$I_A(k,p) = -\frac{1}{2\Delta^2} \left\{ [p^2 - (k \cdot p)] K_0(p-k) + [p^2(k^2 - m^2) - k \cdot p(p^2 - m^2)] I_0(k,p) + p^2 K_0(p) - (k \cdot p) K_0(k) \right\},$$

$$I_B(k,p) = I_A(p,k),$$
(53)

<sup>&</sup>lt;sup>3</sup> Reference [11] develops the method. Based upon it, the 4-point functions were considered (along with the arbitrary number of legs) in [16] for equal masses and in [17] for arbitrary masses.

and  $\Delta^2 = (k \cdot p)^2 - k^2 p^2$ .

# The $\widetilde{I}^{\mu}(k,p)$ Integral:

Similarly, we can write  $\widetilde{I}^{\mu}(k,p)$  as:

$$\widetilde{I}_{\mu}(k,p) = \frac{i\pi^2}{2} [k_{\mu}\widetilde{I}_A(k,p) + p_{\mu}\widetilde{I}_B(k,p)],$$
(54)

where  $\tilde{I}_A(k,p)$  and  $\tilde{I}_B(k,p)$  are given by the same expressions as  $I_A(k,p)$  and  $I_B(k,p)$  in Eqs. (53) with the  $\tilde{I}_B(k,p)$  appearing for each quantity, i.e., with the replacements  $K_0(p-k) \to \tilde{K}_0(p-k)$ ,  $I_0(k,p) \to \tilde{I}_0(k,p)$ ,  $K_0(p) \to \tilde{K}_0(p)$  and  $K_0(k) \to \tilde{K}_0(k)$ .

## The $J^{\mu}(k,p)$ Integral:

In the same way, the most general description of  $J^{\mu}(k,p)$  is:

$$J_{\mu}(k,p) = \frac{i\pi^2}{2} [k_{\mu} J_A(k,p) + p_{\mu} J_B(k,p)] , \qquad (55)$$

where  $J_A(k,p)$  and  $J_B(k,p)$  are given by the same expressions as  $I_A(k,p)$  and  $I_B(k,p)$  respectively in Eqs. (53) with the replacements  $K_0(p-k) \to I_0(k,p)$ ,  $I_0(k,p) \to J_0(k,p)$ ,  $K_0(p) \to L_0(p)$  and  $K_0(k) \to L_0(k)$ .

We now treat the most complicated vector integrals which depend upon three independent momentum variables.

## The $U^{\mu}(p,q,p')$ Integral:

The  $U^{\mu}(p,q,p')$  integral can be expressed only in terms of  $p_{\mu}$ ,  $q_{\mu}$  and  $p'_{\mu}$ . Therefore,

$$U_{\mu}(p,q,p') = \frac{i\pi^2}{2} [p_{\mu}U_A(p,q,p') + q_{\mu}U_B(p,q,p') + p'_{\mu}U_C(p,q,p')], \qquad (56)$$

where

$$U_{A}(p,q,p') = \frac{1}{2[p'^{2}(p\cdot q)^{2} - 2(p\cdot q)(p\cdot p')(p'\cdot q) + q^{2}(p\cdot p')^{2} + p^{2}((p'\cdot q)^{2} - p'^{2}q^{2})]} \times \{[(p\cdot p')(p'\cdot q) - p'^{2}(p\cdot q)]I_{0}(p,p') + [p'^{2}q^{2} - (p'\cdot q)^{2}]I_{0}(p',q) + [(p\cdot q)(p'\cdot q) - q^{2}(p\cdot p')]I_{0}(p,q) + [p'^{2}(p\cdot q) - p'^{2}q^{2} + q^{2}(p\cdot p') - (p\cdot q + p\cdot p')p'\cdot q + (p'\cdot q)^{2}]\widetilde{I}_{0}(q - p, p' - p) + [p'^{2}q^{2}(-p^{2} + p\cdot q + p\cdot p') - (p'\cdot q(p'^{2}(p\cdot q) + q^{2}(p\cdot p')) + p^{2}(p'\cdot q)^{2} + m^{2}(-p'^{2}(p\cdot q) + p'^{2}q^{2} - q^{2}(p\cdot p') + p'\cdot q(p\cdot q + p\cdot p') - (p'\cdot q)^{2})]U_{0}(p,q,p')\},$$

$$U_B(q, p, p') = U_C(p', q, p) = U_A(p, q, p')$$
 (57)

## The $V^{\mu}(p,q,p')$ Integral:

The integral  $V^{\mu}(p,q,p')$  can also be expanded out as follows:

$$V_{\mu} = \frac{i\pi^2}{2} [p_{\mu} V_A(p, q, p') + q_{\mu} V_B(p, q, p') + p'_{\mu} V_C(p, q, p')] , \qquad (58)$$

where  $V_A(p,q,p') = V_B(q,p,p') = V_C(p',q,p)$  are the same as  $U_A(p,q,p') = U_B(q,p,p') = U_C(p',q,p)$  in Eqs. (57) with the following replacements:  $I_0(p,p') \to J_0(p,p')$ ,  $I_0(p',q) \to J_0(p',q)$ ,  $I_0(p,q) \to J_0(p,q)$ ,  $\tilde{I}_0(q-p,p'-p) \to U_0(p,q,p')$  and  $U_0(p,q,p') \to V_0(p,q,p')$ .

## C. Tensor Integrals with Two Indices

These integrals can also be categorized as regards the number of independent momentum variables they depend upon. For only one momentum dependence, we have

$$\widetilde{K}^{\mu\nu}(q) = \frac{1}{4(D-1)} \left[ 2(D-2)T + \left( q^2D - 4m^2 \right) \widetilde{K}(q) \right] \left( \frac{q^{\mu}q^{\nu}}{q^2} - \frac{1}{D}g^{\mu\nu} \right) + \frac{g^{\mu\nu}}{D} \left( T + m^2 \widetilde{K}(q) \right) , \qquad (59)$$

$$L^{\mu\nu}(q) = \frac{1}{4(D-1)q^2} \left[ (q^2 - m^2)^2 L(q) - 2(q^2 + m^2) K(q) + T \right] \left( D \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{q^\mu q^\nu}{q^2} K(q) . \tag{60}$$

Solutions for the integrals with growing complexity have been listed below separately:

## The $I^{\mu\nu}(k,p)$ Integral:

The tensor integral  $I^{\mu\nu}$  can be expressed in terms of the scalar integrals  $K_0, I_C, I_D$  and  $I_E$  as follows:

$$I_{\mu\nu}^{(2)} = \frac{i\pi^2}{2} \left[ \frac{g_{\mu\nu}}{D} K_0(p-k) + \left( k_{\mu} k_{\nu} - g_{\mu\nu} \frac{k^2}{D} \right) I_C + \left( p_{\mu} k_{\nu} + k_{\mu} p_{\nu} - g_{\mu\nu} \frac{2(k \cdot p)}{D} \right) I_D + \left( p_{\mu} p_{\nu} - g_{\mu\nu} \frac{p^2}{D} \right) I_E \right].$$

The coefficients  $I_C$ ,  $I_D$  and  $I_E$  in the above expressions are :

$$I_{C}(k,p) = \frac{D}{(D-2)\Delta^{2}} \left\{ \left[ \left( \frac{1}{2} - \frac{1}{D} \right) (p^{2} - m^{2}) k \cdot p - \left( \frac{1}{2} - \frac{1}{2D} \right) (k^{2} - m^{2}) p^{2} \right] I_{A} - \frac{1}{2D} (p^{2} - m^{2}) p^{2} I_{B}$$

$$+ \left[ \left( \frac{2}{D} - \frac{1}{2} \right) p^{2} + \left( \frac{1}{2} - \frac{1}{D} \right) (k \cdot p) \right] \frac{K_{0}(p-k)}{2} - \left( 1 - \frac{2}{D} \right) p_{\nu} K_{0}^{\nu}(k) \right\},$$

$$I_{D}(k,p) = \frac{D}{2(D-2)\Delta^{2}} \left\{ \left[ (k^{2} - m^{2})k \cdot p - \left( 1 - \frac{2}{D} \right) (p^{2} - m^{2})k^{2} \right] \frac{I_{A}}{2} + \left[ (p^{2} - m^{2})k \cdot p - \left( 1 - \frac{2}{D} \right) (k^{2} - m^{2})p^{2} \right] \frac{I_{B}}{2} - \frac{1}{4} \left[ \left( 1 - \frac{2}{D} \right) (p - k)^{2} + \frac{4}{D} (k \cdot p) \right] K_{0}(p - k)$$

$$+ \left( 1 - \frac{2}{D} \right) \left[ k_{\nu} K_{0}^{\nu}(k) + p_{\nu} K_{0}^{\nu}(p) \right] \right\}.$$

$$(62)$$

Note that the momentum dependence of  $I_A$  and  $I_B$  has not been displayed explicitly. It is  $I_A(k, p)$  and  $I_B(k, p)$ . Moreover,  $I_C(k, p)$  and  $I_E(k, p)$  are symmetric functions under the exchange of k and p. Therefore,

$$I_E(k,p) = I_C(p,k) . (63)$$

# The $J^{\mu\nu}(k,p)$ Integral:

The tensor integral  $J^{\mu\nu}$  has the following decomposition in terms of the scalar integrals  $I_0,J_C,J_D$  and  $J_E$ :

$$J^{(2)}_{\mu\nu} \ = \ \frac{i\pi^2}{2} \bigg[ \frac{g_{\mu\nu}}{D} I_0 + \bigg( k_\mu k_\nu - g_{\mu\nu} \frac{k^2}{D} \bigg) J_C + \bigg( p_\mu k_\nu + k_\mu p_\nu - g_{\mu\nu} \frac{2(k \cdot p)}{D} \bigg) J_D + \bigg( p_\mu p_\nu - g_{\mu\nu} \frac{p^2}{D} \bigg) J_E \bigg] \ .$$

The coefficients  $J_C$ ,  $J_D$  and  $J_E$  in the above expressions are :

$$J_{C}(k,p) = \frac{D}{(D-2)\Delta^{2}} \left\{ \left[ \left( \frac{1}{2} - \frac{1}{D} \right) (p^{2} - m^{2}) k \cdot p - \left( \frac{1}{2} - \frac{1}{2D} \right) (k^{2} - m^{2}) p^{2} \right] J_{A} - \frac{1}{2D} (p^{2} - m^{2}) p^{2} J_{B}$$

$$+ \left[ \left( 1 - \frac{2}{D} \right) k \cdot p - \left( 1 - \frac{1}{D} \right) p^{2} \right] \frac{I_{A}}{2} - \frac{p^{2}}{2D} I_{B} + \frac{p^{2}}{D} I_{0} - \left( 1 - \frac{2}{D} \right) p_{\nu} L_{0}^{\nu}(k) \right\} , \qquad (64)$$

$$J_{D}(k,p) = \frac{D}{2(D-2)\Delta^{2}} \left\{ \left[ (k^{2} - m^{2}) k \cdot p - \left( 1 - \frac{2}{D} \right) (p^{2} - m^{2}) k^{2} \right] \frac{J_{A}}{2} + \left[ (p^{2} - m^{2}) k \cdot p - \left( 1 - \frac{2}{D} \right) (k^{2} - m^{2}) p^{2} \right] \frac{J_{B}}{2} + \left[ k \cdot p - \left( 1 - \frac{2}{D} \right) k^{2} \right] \frac{I_{A}}{2} + \left[ k \cdot p - \left( 1 - \frac{2}{D} \right) p^{2} \right] \frac{I_{B}}{2}$$

$$- \frac{2}{D} (k \cdot p) I_{0} + \left( 1 - \frac{2}{D} \right) \left[ k_{\nu} L_{0}^{\nu}(k) + p_{\nu} L_{0}^{\nu}(p) \right] \right\} . \qquad (65)$$

The momentum dependence of  $I_A$ ,  $I_B$ ,  $J_A$ ,  $J_B$  is  $I_A(k,p)$ ,  $I_B(k,p)$ ,  $J_A(k,p)$ ,  $J_B(k,p)$ . Moreover, as is evident,  $J_C(k,p)$  and  $J_E(k,p)$  are symmetric under the exchange of k and k, i.e.,

$$J_E(k,p) = J_C(p,k) . (66)$$

The  $U^{\mu\nu}(p,q,p')$  Integral:

The general expansion of  $U^{\mu\nu}$  in terms of the scalar integrals  $\widetilde{I}_0, U_D, U_E, U_F, U_G, U_H$  and  $U_I$  is as follows:

$$U_{\mu\nu}(p,q,p') = \frac{i\pi^2}{2} \left[ \frac{g_{\mu\nu}}{D} \widetilde{I}_0(q-p,p'-p) + \left( p_{\mu}p_{\nu} - g_{\mu\nu}\frac{p^2}{D} \right) U_D + \left( p_{\mu}q_{\nu} + q_{\mu}p_{\nu} - g_{\mu\nu}\frac{2(p \cdot q)}{D} \right) U_E \right. \\ \left. + \left( p_{\mu}p'_{\nu} + p'_{\mu}p_{\nu} - g_{\mu\nu}\frac{2(p \cdot p')}{D} \right) U_F + \left( q_{\mu}q_{\nu} - g_{\mu\nu}\frac{q^2}{D} \right) U_G \right. \\ \left. + \left( q_{\mu}p'_{\nu} + p'_{\mu}q_{\nu} - g_{\mu\nu}\frac{2(p' \cdot q)}{D} \right) U_H + \left( p'_{\mu}p'_{\nu} - g_{\mu\nu}\frac{p'^2}{D} \right) U_I \right],$$

$$(67)$$

where

$$U_{D}(p,q,p') = \frac{1}{2(D-3)\left[p^{2}(p'\cdot q)^{2} - 2(p\cdot p')(p\cdot q)(p'\cdot q) + (p\cdot p')^{2}q^{2} + p'^{2}\left((p\cdot q)^{2} - p^{2}q^{2}\right)\right]} \\ \left\{ \left[ (D-2)p^{2}(p'\cdot q)^{2} - (D-3)(p'^{2}p\cdot q + p\cdot p'q^{2})p'\cdot q + p'^{2}((D-3)(p\cdot p' + p\cdot q) - (D-2)p^{2})q^{2} + m^{2}(-(D-2)(p'\cdot q)^{2} + (D-3)(p\cdot p' + p\cdot q)p'\cdot q - (D-3)p'^{2}p\cdot q + (D-2)p'^{2}q^{2} - (D-3)p\cdot p'q^{2})\right]U_{A}(p,q,p') \\ + \left[ (m^{2} - q^{2})\left(p'^{2}q^{2} - (p'\cdot q)^{2}\right)\right]U_{B}(p,q,p') \\ + \left[ (p'^{2} - m^{2})\left((p'\cdot q)^{2} - p'^{2}q^{2}\right)\right]U_{C}(p,q,p') \\ + \left[ (D-4)(p'\cdot q)^{2} - (D-3)(p\cdot p' + p\cdot q)p'\cdot q + (D-3)p'^{2}p\cdot q - (D-4)p'^{2}q^{2} + (D-3)p\cdot p'q^{2}\right]\widetilde{I}_{0}(q-p,p'-p) \\ - (D-3)\left[ (p'\cdot q)^{2} - (p\cdot p' + p\cdot q)p'\cdot q + p'^{2}(p\cdot q - q^{2}) + p\cdot p'q^{2}\right] \times \\ \left[ \widetilde{I}_{A}(q-p,p'-p) + \widetilde{I}_{B}(q-p,p'-p) \right] + (D-3)\left[ (p'\cdot q)(p\cdot p') - p'^{2}p\cdot q \right] \times \\ I_{A}(p,p') + (D-3)\left[ (p'\cdot q)(p\cdot q) - p\cdot p'q^{2}\right]I_{A}(p,q) \right\},$$
(68)

$$U_{H}(p,q,p') = \frac{1}{2(D-3) \left[p^{2}(p'\cdot q)^{2} - 2(p\cdot p')(p\cdot q)(p'\cdot q) + (p\cdot p')^{2}q^{2} + p'^{2}((p\cdot q)^{2} - p^{2}q^{2})\right]}$$

$$\left\{ - \left[ (m^{2} - p^{2})((p'\cdot q)p^{2} - (p\cdot p')(p\cdot q))\right] U_{A}(p,q,p') \right.$$

$$\left. + \left[ (D-3)p\cdot q(p'^{2}p\cdot q - p'\cdot qp^{2}) + (-p^{2}((D-3)p'^{2} + p'\cdot q + 3p\cdot p' - D(p'\cdot q + p\cdot p')) - (D-1)p\cdot p'p\cdot q)q^{2} + m^{2}(p'\cdot q(-Dp^{2} + p^{2} + (D-3)p\cdot q) - p\cdot q(-Dp\cdot p' + p\cdot p' + (D-3)p\cdot q) + (D-3)(p^{2} - p\cdot p')q^{2})\right]$$

$$\frac{U_{B}(p,q,p')}{2} + \left[ m^{2}(p'\cdot q(-Dp^{2} + p^{2} + (D-3)p\cdot p') + (D-3)p'^{2}(p^{2} - p\cdot q) - p\cdot p'((D-3)p\cdot p' - Dp\cdot q + p\cdot q)) + p'^{2}((D-1)p'\cdot qp^{2} + (D-3)(p\cdot q - q^{2})p^{2} - (D-1)p\cdot p'p\cdot q) + (D-3)p\cdot p'(p\cdot p'q^{2} - p'\cdot qp^{2})\right]$$

$$\frac{U_{C}(p,q,p')}{2} + \left[ (p\cdot p')(p\cdot q) - p'\cdot qp^{2}\right] \widetilde{I_{0}}(q-p,p'-p)$$

$$+ \frac{D-3}{2} \left[ p\cdot q^{2} - p'\cdot qp\cdot q - p\cdot p'p\cdot q + p'\cdot qp^{2} - p^{2}q^{2} + p\cdot p'q^{2} \right]$$

$$\widetilde{I_{A}}(q-p,p'-p) + \frac{D-3}{2} \left[ p'\cdot q(p^{2} - p\cdot p') + p\cdot p'(p\cdot p' - p\cdot q) + p'^{2}(p\cdot q-p^{2})\right] \widetilde{I_{B}}(q-p,p'-p) + \frac{D-3}{2} \left[ p'^{2}p^{2} - (p\cdot p')^{2}\right] I_{B}(p,p')$$

$$+ \frac{D-3}{2} \left[ p^{2}q^{2} - (p\cdot q)^{2}\right] I_{B}(p,q) + \frac{D-3}{2} \left[ p'\cdot qp\cdot q - p\cdot p'q^{2}\right] I_{A}(p',q)$$

$$+\frac{D-3}{2}[p'\cdot qp\cdot p'-p'^{2}p\cdot q]I_{B}(p',q)\}.$$
(69)

$$U_G(q, p, p') = U_I(p', q, p) = U_D(p, q, p'), \quad U_E(p, q, p') = U_H(p', q, p) \quad U_F(p, q, p') = U_H(q, p, p'). \tag{70}$$

The  $V^{\mu\nu}(p,q,p')$  Integral:

Similarly to the previous case, the  $V^{\mu\nu}$  integral can be expanded in terms of  $U_0, V_D, V_E, V_F, V_G, V_H$  and  $V_I$  as follows:

$$V_{\mu\nu}^{(2)}(p,q,p') = \frac{i\pi^2}{2} \left[ \frac{g_{\mu\nu}}{D} U_0(p,q,p') + \left( p_{\mu} p_{\nu} - g_{\mu\nu} \frac{p^2}{D} \right) V_D + \left( p_{\mu} q_{\nu} + q_{\mu} p_{\nu} - g_{\mu\nu} \frac{2(p \cdot q)}{D} \right) V_E \right. \\ \left. + \left( p_{\mu} p_{\nu}' + p_{\mu}' p_{\nu} - g_{\mu\nu} \frac{2(p \cdot p')}{D} \right) V_F + \left( q_{\mu} q_{\nu} - g_{\mu\nu} \frac{q^2}{D} \right) V_G \right. \\ \left. + \left( q_{\mu} p_{\nu}' + p_{\mu}' q_{\nu} - g_{\mu\nu} \frac{2(p' \cdot q)}{D} \right) V_H + \left( p_{\mu}' p_{\nu}' - g_{\mu\nu} \frac{p'^2}{D} \right) V_I \right], \tag{71}$$

where the independent scalar integrals are :

$$V_{D}(p,q,p') = \frac{1}{2(D-3)\left[p^{2}(p'\cdot q)^{2} - 2(p\cdot p')(p\cdot q)(p'\cdot q) + (p\cdot p')^{2}q^{2} + p'^{2}((p\cdot q)^{2} - p^{2}q^{2})\right]}$$

$$\left\{ \left[ (D-2)p^{2}(p'\cdot q)^{2} - (D-3)(p'^{2}p\cdot q + p\cdot p'q^{2})p'\cdot q + p'^{2}((D-3)(p\cdot p' + p\cdot q) - (D-2)p^{2})q^{2} + m^{2}((D-2)(p'\cdot q)^{2} + (D-3)(p\cdot p' + p\cdot q)p'\cdot q - (D-3)p'^{2}p\cdot q + (D-2)p'^{2}q^{2} - (D-3)p\cdot p'q^{2})\right]V_{A}(p,q,p')$$

$$+ \left[ (m^{2} - q^{2})(p'^{2}q^{2} - (p'\cdot q)^{2})\right]V_{B}(p,q,p') + \left[ (p'^{2} - m^{2})((p'\cdot q)^{2} - p'^{2}q^{2})\right]$$

$$V_{C}(p,q,p') + \left[ (D-2)(p'\cdot q)^{2} - (D-3)(p\cdot p' + p\cdot q)p'\cdot q + (D-3)p'^{2}p\cdot q - (D-2)p'^{2}q^{2} + (D-3)p\cdot p'q^{2}\right]U_{A}(p,q,p') + \left[ (p'\cdot q)^{2} - p'^{2}q^{2}\right]\left[U_{B}(p,q,p') + (U_{C}(p,q,p'))\right] + (D-3)\left[p'\cdot qp\cdot p' - p'^{2}p\cdot q\right]J_{A}(p,p')$$

$$+ (D-3)\left[p'\cdot qp\cdot q - p\cdot p'q^{2}\right]J_{A}(p,q) + 2\left[p'^{2}q^{2} - (p'\cdot q)^{2}\right]U_{0}(p,q,p')\right\},$$

$$(72)$$

$$V_{H}(p,q,p') = \frac{1}{2(D-3)[p^{2}(p'\cdot q)^{2} - 2(p\cdot p')(p\cdot q)(p'\cdot q) + (p\cdot p')^{2}q^{2} + p'^{2}((p\cdot q)^{2} - p^{2}q^{2})]} \left\{ -[(m^{2}-p^{2})(p'\cdot qp^{2} - p\cdot p'p\cdot q)]V_{A}(p,q,p') + [(D-3)p\cdot q(p'^{2}p\cdot q - p'\cdot qp^{2}) + (-p^{2}((D-3)p'^{2} + p'\cdot q + 3p\cdot p' - D(p'\cdot q + p\cdot p')) - (D-1)p\cdot p'p\cdot q)q^{2} + m^{2}(p'\cdot q(-Dp^{2} + p^{2} + (D-3)p\cdot q) - p\cdot q(-Dp\cdot p' + p\cdot p' + (D-3)p\cdot q) + (D-3)(p^{2} - p\cdot p')q^{2})]\frac{V_{B}(p,q,p')}{2} \left[ m^{2}(p'\cdot q(-Dp^{2} + p^{2} + (D-3)p\cdot p' + (D-3)p'^{2}(p^{2} - p\cdot q) - p\cdot p'((D-3)p\cdot p' - Dp\cdot q + p\cdot q)) + p'^{2}((D-1)p'\cdot qp^{2} + (D-3)(p\cdot q - q^{2})p^{2} - (D-1)p\cdot p'p\cdot q) + (D-3)p\cdot p' + (D-3)p'^{2}(p\cdot q - p^{2}) + p\cdot p'((D-3)p\cdot p' - Dp\cdot q + p\cdot q)]\frac{U_{C}(p,q,p')}{2} + (D-3)^{2}[p'^{2}p^{2} - (p\cdot p')^{2}]J_{B}(p,p') + \frac{(D-3)}{2}[p^{2}q^{2} - (p\cdot q)^{2}]J_{B}(p,q) + \frac{(D-3)}{2}[p'\cdot qp\cdot q - p\cdot p'\cdot qp^{2}]J_{A}(p',q) + \frac{(D-3)}{2}[p'\cdot qp\cdot p' - p'^{2}p\cdot q]J_{B}(p',q) + 2[p\cdot p'p\cdot q - p'\cdot qp^{2}]U_{0}(p,q,p')\},$$
(73)

$$V_G(q, p, p') = V_I(p', q, p) = V_D(p, q, p') \quad V_E(p, q, p') = V_H(p', q, p) \quad V_F(p, q, p') = V_H(q, p, p'). \tag{74}$$

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